Warm up:

1. CST

What is the solution to this system of equations?

$$\begin{cases} y = -3x - 2 \\ 6x + 2y = -4 \end{cases}$$

A (6, 2)

B (1, –5)

C no solution

D infinitely many solutions

- Why does it not work to plug in the values?

2. Review

Solve: $x + y = 7$, $3x + 7y = 29$

- State how to check your answer and then check it.

3. Current

Do not solve the problem. Define the unknown quantities by choosing a variable and using a complete sentence to describe each unknown quantity.

A library is having a book sale to raise money. Hardcover books cost $4 each and paperback books cost $2 each. A person spends $26 for 8 books. How many hardcover books did she purchase?

• What two things do you need to know in order to state how much she spent on just the hardcover books. Is anything already given?

4. Other

Fill in the guess and check chart for problem #3 to get the answer.

Hardback Book's Total cost	Paperback Book's Total cost	Total Cost of Purchase
$0 \bullet \$4 = \$0$	$8 \bullet \$2 = \$16$	$16
1	7	
2	6	
3	5	
4	4	
5	3	
6	2	
7	1	
8	0	

Discussion & Manipulative:

Start by going over the rules of using manipulative:

"Today we are going to be using little pieces called "mosaics". They can easily make a mess, so what I would like to do is start by telling you the rules, so you can focus on your learning."

1. Do not make a mess!
   - If you are caught doing anything that makes a mess you will lose your privilege and I will take the manipulative.
2. Keep your bag organized and do not mix any other colors with any other bag.
3. Keep all cups in tact and give me back the materials exactly the way you received them.
4. You will turn in your cups stacked, bag of mosaics sealed, mat and overhead pen as your ticket out the door.

Pass out cups and bags of different colored objects? Give students 30 seconds to play without making a mess.

Tell students, "You are in a candy store. Some of you may be hungry for a lot of candy and others may not. Put as much candy in one of your cups as you are hungry for. The color objects represent the candy. You may also mix the colors or "candy" in the cup, it doesn't matter."

- Hopefully this will encourage students to put different amounts.

Then ask students, "Partner with the person next to you. Who will pay more for their candy? You or them? Give a reason why, making statements about how much you will pay."

- You should hear comments like I have more, so I will pay more and vice versa.

Now tell students, "Were you able to say who paid more? Unless candy all cost the same, nobody should have been able to state who paid more. Now, the person on the right bought expensive chocolate that costs $5 a pound and the person on the left's candy only costs $1 a pound, since it is on sale. Now who will pay more for their candy? Again make statements about your own cost."

- You should hear things like mine cost more because I have more and it cost more. Also, you should have some students who are not sure because they have a little but it costs a more, but the other students has a lot more than them. Let them hash it out with your assistance. Maybe make it a whole group discussion to help decide who will pay more.

Now say, "What two things determine how much it will cost you to buy candy in a candy store?"

- Students should be able to say the amount you have or weight and how much it cost per unit, they will probably talk about pounds. Also discuss other measurements of weights and symbols like: $oz, lb, t, mg, g,$ and kg or ounce, pound, ton, milligram, gram and kilogram.

Notes: (Have students write the following in their notes.)

"When determining total cost you need the weight and the cost per unit."

Total cost = weight • Price
Possible weights are: ounce = oz, pound = lb, ton = t, milligram = mg, gram = g and kilogram = kg.

Discussion & Manipulative: (Hand out the mats)

Tell Students, "Carefully, dump your candy back into the bags. Now put 3 of one color in one cup and 5 of the other color in a second cup. Place the two cups on the first two circles on the mat. What we are saying is that we have 3 *lbs* of one candy and we are adding 5 *lbs* of another candy to it, so each piece weighs 1 *lbs*. Place the amount of candy in the 3rd cup to make it equal to mixing the 1st and 2nd cup together. If we mix these two candies together, how many total *lbs* will we have in the third cup? 8 *lbs* Now lets try a mixture problem."

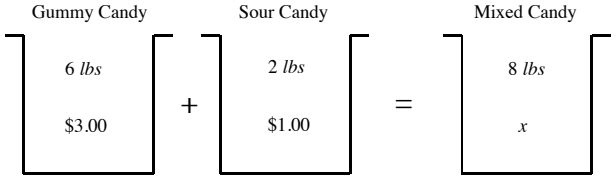
Notes: (Have students write the following in their notes.)

Example 1: A candy storeowner wants to make a mix of sour and gummy candy, since he has a surplus of gummy candies and is low on sour candies. He wants to mix 6 *lbs* of gummy candy costing $3.00 a pound with 2 *lbs* of sour candy costing $1.00. What will the cost per pound be of the mix?

Say, "What two things are you mixing? What do you need to know in order to find the total cost for each candy and for the mixture's total cost? What is the unknown quantity? Help me fill in a cup chart with the information we have from the problem."

Define your variable: Let $x =$ the cost per pound of the mix

Explain how to fill in the title of each cup and the two parts you need to find the total cost. Talk about each quantity and where it goes in the chart or cups below. Talk about how many unknowns there are and how this problem is the simplest one because there is only one unknown. Then solve explaining how to get the equation.



$$6(3.00)+2(1.00)=8(x)$$
$$6(3)+2(1)=8(x)$$
$$`18+2=8x$$
$$20=8x$$
$$x=2.5$$

The mixture will be $2.50 per pound.

(If you have time) **YOU TRY**: A farm stand owner mixes apple juice and cranberry juice. How much should he charge if he mixes 8 L of apple juice selling for $\$0.45/L$ with 10 L of cranberry juice selling for $\$1.08/L$?

Discussion & Manipulative:

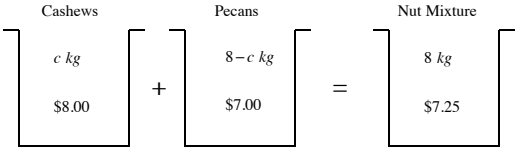
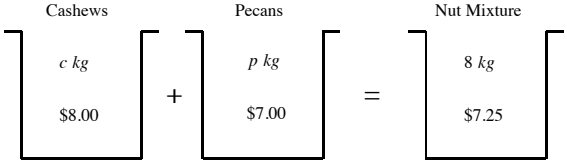
Say, "Now lets talk about knowing different information. What if we know that we have a mixture of two types of candy, M&M's and Skittles, that totals $9\,lbs$, do we know many pounds of each candy we have? What could we call one of the candies? Lets define a variable for the M&M's Let $m =$ the number of pounds of the M&M's. How do you define the number of pounds of the Skittles? We could do it in two ways. The first way we will do it is by using one variable and the second way is using two variables.

One Variable	vs.	Two Variables
"In your third cup put the total pounds of the mixture $9\,lbs$. In order to know how many pounds of gummy worms we have in the mixture, we need to take out all the pounds of jawbreakers. Mathematically, what does take out mean? If you have $9\,lbs$ and we take out the pounds of jawbreakers we will have the number of pounds of gummy worms. Since, I defined the number of pounds of jawbreakers to be m, I need to translate," $9\,lbs$ taking out m pounds to be $9-m$. "We now know that," $m =$ the number of pounds of the M & M's $9-m =$ the number of pounds of the Skittles		"Since we do not know how many pounds of jawbreakers or how many pounds of gummy worms we have, there are two unknown things." In this case I will use two variables and I need two equations to solve a system of equations. We will let $m =$ the number of pounds of the M & M's $s =$ the number of pounds of the Skittles

Notes: (Have students write the following in their notes.)

Example 2: The owner of Fancy Food Shoppe wants to mix cashews selling at $\$8.00/kg$ and pecans selling at $\$7.00/kg$. How many kilograms of each kind of nut should be mixed to get $8\ kg$ worth $\$7.25/kg$?

Say, "What two things are you mixing? What do you need to know in order to find the total cost for each nut and for the mixture? What is the unknown quantity? Help me fill in a cup chart with the information we have from the problem."

One Variable	vs.	Two Variables
Define a variable: Let $c =$ the number of kg for cashews. Translate: $8\,kg$ taking out c kilograms to be $8-c$. We now know that, $c =$ the number of kilograms of cashews $8-c =$ the number of kilograms of pecans • Explain how to fill in the title of each cup and the two parts you need to find the total cost. Talk about each quantity and where it goes in the chart or cups below. Explain how there is only one unknown and there will only be one equation. Then solve the problem explaining how to get the equation.  • Show how you can add across the kilogram line to check to see if they add to 8 kilograms. **Remember that Total cost = Weight * Price**		"Since we do not know how many kilograms of cashews or pecans we have, there are two unknown things." In this case I will use two variables to solve a system of equations. We will let $c =$ the number of kilograms of cashews $p =$ the number of kilograms of pecans • Explain how to fill in the title of each cup and the two parts you need to find the total cost. Talk about each quantity and where it goes in the chart or cups below. Talk about how many unknowns there are and how this problem will have two equations. Talk about possible letters to use for the variables. Then solve explaining how to get the equations.  • Show how you can add across the pound line to get an equation, but cannot do it for the dollars. You have to talk about total cost of each item, which consists of two things weight times price.

$$8(c)+7(8-c)=8(7.25)$$
$$8c+56-7c=58$$
$$c+56=58$$
$$c+56-56=58-56$$
$$c=2$$

"We defined"
$c=$ the number of kilograms of cashews
$8-c=$ the number of kilograms of pecans
$\therefore$ There are 2 *kg* of cashews and 8 – 2 or 6 *kg* of pecans.

Equation 1: $c+p=8$
Equation 2: $c(8)+p(7)=8(7.25)$
$8c+7p=58$

"Solve the system by using substitution and solve for p in equation 1"
$$p=8-c$$
"Substitute for p in equation 2" $8c+7(8-c)=58$
"Notice this equation is the same as the one we used for one variable."
$$\therefore c=2$$
"We defined" $c=$ the number of kilograms of *c*ashews
$p=$ the number of kilograms of pecans
"We still need to find p, so plug in 2 for c to find p."
$$p=8-2$$
$$p=6$$
$\therefore$ There are 2 *kg* of cashews and 6 *kg* of pecans.

YOU TRY: Use whichever method you prefer: Joanne makes a mixture of dried fruits by mixing dried apples costing $\$6.00/kg$ with dried apricots costing $\$8.00/kg$. How many kilograms of each are needed to make $20\ kg$ of mixture worth $\$7.20/kg$?

Discussion & Manipulative:

"Now we are going to discuss mixing liquids that has a percent of acid, juice, copper, etc.. There are three situations"
1. The problem will give you all 3 different percents for the original, what you are adding and the mix.
2. The problem will decrease the original liquid by diluting it.
3. The problem will increase the potency of the original liquid.

Lets talk about #2 and #3 because these you have to add a percent not given to you. How do you think you can dilute a liquid? What could you add to dilute something?" You could say the following if no one guesses water, "What would you add to lemonade if it was too strong?" Water. "Okay, what would you add to lemonade if the final mix was too week in taste?" Lemon juice.

"Now, when we talk about diluting or increasing liquids we state what percent each item consists of and the amount of the liquid. For lemonade I would say that I have $2\ L$ of Lemonade that is 30% lemon juice. If I wanted to increase the potency, what would I add to the lemonade? Lemon juice. What percent of lemon juice is lemon juice? 100%. What if I wanted to dilute it, what would I add to the lemonade? Water. What percent of lemon juice is in water? 0%."

Notes: (Have students write the following in their notes.)

When mixing liquids you have 3 cases:
1. The problem will give you all 3 different percents for the original, what you are adding and the mix.
2. The problem will decrease the original liquid by diluting it.
   - Dilute by adding water: Since water has nothing else in it, the percent will be 0.
3. The problem will increase the potency of the original liquid.
   - Increase the potency by adding a pure ingredient: Since you adding a pure ingredient, the percent is 100.

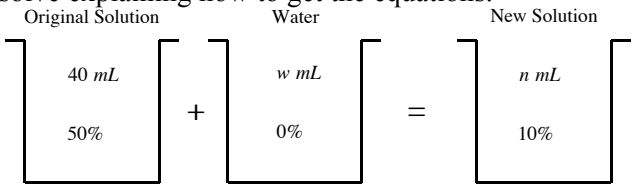
Also notice you will have different units: $L=$ Liter, $mL=$ milliliter, $pt=$ pint, $qt=$ quart, or $gal=$ gallon

Example 3: A chemist has $40\ mL$ of a solution that is 50% acid. How much water should be added to make a solution that is 10% acid?

"Are we increasing the potency or diluting the liquid? Diluting. What does the % represent? Acid. Is there any acid in water? No. Then what percent acid does water have? 0%. Lets define the variable & fill in a chart."

Define the variables:
"What is it asking us to find? How much water?" Talk about possible letters to use for the variables.
Let w = the number of mL of water "What else is unknown? The amount of the new solution."
Let n = the number of mL of the new solution

Explain how to fill in the title of each cup and the two parts you need to find the total amount of acid. Talk about each quantity and where it goes in the chart below. Talk about how many unknowns there are and how this problem will have two equations. Then solve explaining how to get the equations.



- Show how you can add across the milliliter line to get an equation, but cannot do it for the percents. You have to talk about total amount of acid of each liquid, which consists of two things total amount times percent of acid.

Equation 1: $40 + w = n$

Equation 2: $40\left(\frac{50}{100}\right) + w\left(\frac{0}{100}\right) = n\left(\frac{10}{100}\right)$

"For equation 2 simplify fractions first, then clear the denominator"

$$20 = \frac{1}{10}n$$

$$10(20) = 10\left(\frac{1}{10}n\right)$$

"Solve the system using the substitution method"

1) $40 + w = n$
2) $200 = n$

"Substitute for n in equation 2), since it is already solved for" $200 = 40 + w$
"Solve for w" $w = 160$

You need to add 160 mL of water.

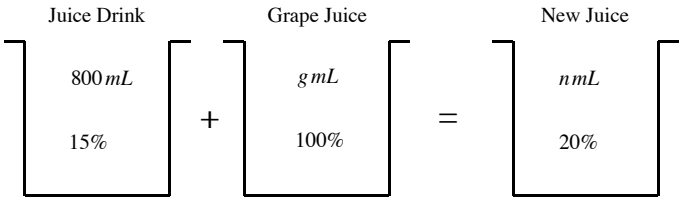
(If you have time) **YOU TRY**: How many liters of water must be added to $20\,L$ of a 24% acid solution to make a solution that is 8% acid?

Example 4: If $800\ mL$ of a juice drink is 15% grape juice, how much grape juice should be added to make a drink that is 20% grape juice?

"Are we increasing the potency or diluting the liquid? Increasing. What does the % represent? Grape juice. What percent grape juice does pure grape juice have? 100%. Lets define the variable and fill in a chart."

Define the variables:
"What is it asking us to find? How much grape juice should be added?" Talk about possible letters to use for the variables.
Let g = the number of mL of pure grape juice "What else is unknown? The amount of the new drink."
Let n = the number of mL of the new juice

Explain how to fill in the title of each cup and the two parts you need to find the total amount of grape juice in each drink. Talk about each quantity and where it goes in the chart below. Talk about how many unknowns there are and how this problem will have two equations. Then solve explaining how to get the equations.



- Show how you can add across the milliliter line to get an equation, but cannot do it for the percents. You have to talk about total amount of grape juice of each liquid, which consists of two things total amount of juice times percent of grape juice.

Equation 1: $800 + g = n$

Equation 2: $800\left(\frac{15}{100}\right) + g\left(\frac{100}{100}\right) = n\left(\frac{20}{100}\right)$

"For equation 2 simplify fractions first, then clear the denominator."

$120 + g = \frac{1}{5}n$

$5(120 + g) = 5\left(\frac{1}{5}n\right)$

"Solve the system using the substitution method"

1) $800 + g = n$
2) $600 + 5g = n$

"Substitute equation 1 for n in equation 2)" $600 + 5g = 800 + g$

"Solve for g" $4g = 200$

$g = 50$

You need to add 50 mL of grape juice.

YOU TRY: *How many kilograms of a 45% copper alloy must be added to $62\,kg$ of a 60% copper alloy to form an alloy, which is 50% copper?

*Note: This is the type of percent problem that gives all 3 percents. You may need to set up this problem for some students.